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Generalized dual-phase lag bioheat equations based on nonequilibrium heat transfer in living biological tissues

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ABSTRACT

Based on a nonequilibrium heat transfer model in the living tissue obtained by performing volume average to the local instantaneous energy equations for blood and tissues, the dual-phase lag bioheat equations with blood or tissue temperature as sole unknown temperature are obtained by eliminating the tissue or blood temperature from the nonequilibrium model. The present dual-phase model successfully overcame the drawbacks of the existing dual-phase lag bioheat equation obtained by simply modifying the classical Pennes bioheat equation. Under the dual-phase model developed in this work, the phase lag times are expressed in terms of the properties of blood and tissue and the interphase convective heat transfer coefficient and blood perfusion rate. The phase lag times for heat flux and temperature gradient for the living tissue are estimated using the available properties from the literature. It is found that the phase lag times for heat flux and temperature gradient for the living tissue are very close to each other.

1. Introduction

Heat transfer in living tissue is accompanied by metabolic heat generation and blood perfusion. The former results from a series of chemical reactions occurring in the living cells; while the latter involves energy exchange between the living tissue and blood flowing through small capillaries in the tissue. During hyperthermia therapy in which external heat source is applied to the tissue, the bioheat equation [1] can be expressed as

$$\rho_s c_s \frac{\partial T_s}{\partial t} = \nabla \cdot (k \nabla T_s) + w_b c_b (T_b - T_s) + S_m + S \tag{1}$$

where S_m and S are the source terms due to metabolic heating and hyperthermia therapy. Eq. (1) is one of the earliest bioheat equations that describes the temperature in a living tissue. The artery blood temperature, T_b, was assumed to be uniform throughout the tissue and the vein blood temperature was assumed to be equal to the tissue temperature. In addition, the blood perfusion effect is assumed to be homogeneous and isotropic, i.e., the effect of directional blood flow cannot be described by the Pennes bioheat equation. Although the Pennes bioheat equation is questionable, it has been applied with great success as an analytical tool with which blood perfusion rate can be determined from experimentally measured local temperature gradients and heat flows. Since the Pennes bioheat equation was proposed, it has been referred in nearly all works in the area of bioheat transfer.

In addition to the drawbacks discussed above, the Pennes bioheat equation is also based the classical Fourier's law of heat conduction:

$$\mathbf{q}(\mathbf{r}, t) = -k \nabla T(\mathbf{r}, t) \tag{2}$$

which assumes that thermal disturbance propagates with an infinite speed. As heat conduction in the biological tissue is accomplished by interaction between the blood and the tissue, the prorogation of thermal disturbance is always at a finite speed. For heat transfer in biological materials with nonhomogeneous inner structures, heat flux equilibrates to the imposed temperature gradient via a relaxation mechanism [2–4]. To incorporate such a non-traditional mechanism, hyperbolic thermal wave models [5–7] have been proposed. In this methodology, the Fourier's law is replaced by:

$$\mathbf{q}(\mathbf{r}, t) + \tau \frac{\partial \mathbf{q}(\mathbf{r}, t)}{\partial t} = -k \nabla T(\mathbf{r}, t) \tag{3}$$

where τ is the thermal relaxation time. Kaminski [3] suggested that the thermal relaxation time for biological systems is in the range of 20–30 s. Mitra et al. [4] experimentally measured the thermal relaxation time and reported that τ for a processed meat was approximately 16 s. These values are very high and have been criticized by many researchers. If Eq. (3) is used to replace Eq. (2) in derivation of Pennes bioheat equation, the following bioheat equation is obtained:

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Nomenclature

a_b	specific heat transfer area [m^2/m^3]
c_b	specific heat of blood [J/(kg K)]
c_s	specific heat of tissues [J/(kg K)]
d_b	diameter of the blood vessel [m]
G	coupling factor between blood and tissue [$W/(m^3 K)$]
h_a	heat transfer coefficient inside the blood vessel [$W/(m^2 K)$]
k_b	thermal conductivity of blood [W/(m K)]
k_s	thermal conductivity of tissue [W/(m K)]
Nu	Nusselt number
$\mathbf{q}''$	heat flux vector [W/m^2]
$\mathbf{r}$	position vector [m]
S	heat source due to hyperthermia therapy [W/m^3]
S_m	source terms due to metabolic heating [W/m^3]
t	time [s]
T_b	blood temperature [K]
T_s	tissue temperature [K]
$\mathbf{V}$	intrinsic phase averaged blood velocity vector [m/s]
w_b	blood perfusion rate [kg/s]

Greek symbols

α	thermal diffusivity [m^2/s]
ε	porosity
ρ_b	blood mass density [kg/m^3]
ρ_s	tissue density [kg/m^3]
τ	thermal relaxation time in hyperbolic model [s]
τ_q	phase lag time of the heat flux [s]
τ_T	phase lag of the temperature gradient [s]

Subscripts

b	blood
eff	effective
s	solid matrix (tissue)

$$\tau\frac{\partial^2 T_s}{\partial t^2}+\left(1+\frac{w_b c_b}{\rho c}\tau_q\right)\frac{\partial T_s}{\partial t}=\alpha_s\nabla^2 T_s+\frac{w_b c_b}{\rho_s c_s}(T_b-T_s)+\frac{w_b c_b \tau}{\rho_s c_s}\frac{\partial T_b}{\partial t}+\frac{S_m+S}{\rho c}+\frac{\tau}{\rho_s c_s}\left(\frac{\partial S_m}{\partial t}+\frac{\partial S}{\partial t}\right) \quad (4)$$

which is referred to as hyperbolic bioheat equation [7]. In laser hyperthermia, laser coagulation, and laser surgery, the thermal effect of the lasers are employed. If the pulse width of the laser is shorter than the thermal relaxation time, the hyperbolic effect must be taken into account.

Eq. (3) can be viewed as the first order approximation of the following equation:

$$\mathbf{q}''(\mathbf{r},t+\tau)=-k\nabla T(\mathbf{r},t) \quad (5)$$

which indicates that there is a delay between the heat flux vector and the temperature gradient. For the same point in the conduction medium, the temperature gradient is established at time t, but the heat flux vector will be established at a later time $t+\tau$, i.e., the relaxation time, τ, can be interpreted as the time delay from the onset of the temperature gradient to the heat flux vector. While the thermal wave model assumes that the temperature gradient always precedes the heat flux, Tzou [8] proposed a dual-phase lag (DPL) model that allows either the temperature gradient (cause) to precede heat flux vector (effect) or the heat flux vector (cause) to precede the temperature gradient (effect), i.e.,

$$\mathbf{q}''(\mathbf{r},t+\tau_q)=-k\nabla T(\mathbf{r},t+\tau_t) \quad (6)$$

where τ_q is the phase lag for the heat flux vector, while τ_T is the phase lag for the temperature gradient. If $\tau_q > \tau_T$, the *local* heat flux vector is the result of the temperature gradient at the same location but an early time. On the other hand, if $\tau_q < \tau_T$, the temperature gradient is the result of the heat flux at an early time. The first order approximation of Eq. (6) is:

$$\mathbf{q}''+\tau_q\frac{\partial \mathbf{q}''}{\partial t}=-k\left[\nabla T+\tau_T\frac{\partial}{\partial t}(\nabla T)\right] \quad (7)$$

If Eq. (7) is used to replace the Fourier's law of conduction, the Pennes bioheat equation becomes [9,10]:

$$\tau_q\frac{\partial^2 T_s}{\partial t^2}+\left(1+\frac{w_b c_b}{\rho_s c_s}\tau_q\right)\frac{\partial T_s}{\partial t}=\alpha_s\left[\nabla^2 T_s+\tau_T\frac{\partial}{\partial t}(\nabla^2 T_s)\right]+\frac{w_b c_b}{\rho_s c_s}(T_b-T_s)+\frac{w_b c_b \tau_q}{\rho_s c_s}\frac{\partial T_b}{\partial t}+\frac{S_m+S}{\rho_s c_s}+\frac{\tau_q}{\rho_s c_s}\left(\frac{\partial S_m}{\partial t}+\frac{\partial S}{\partial t}\right) \quad (8)$$

Under the assumption of constant blood temperature which the Pennes equation is based upon, Eq. (8) becomes:

$$\tau_q\frac{\partial^2 T_s}{\partial t^2}+\left(1+\frac{w_b c_b}{\rho_s c_s}\tau_q\right)\frac{\partial T_s}{\partial t}=\alpha_s\left[\nabla^2 T_s+\tau_T\frac{\partial}{\partial t}(\nabla^2 T_s)\right]+\frac{w_b c_b}{\rho_s c_s}\times(T_b-T_s)+\frac{S_m+S}{\rho_s c_s}+\frac{\tau_q}{\rho_s c_s}\left(\frac{\partial S_m}{\partial t}+\frac{\partial S}{\partial t}\right) \quad (9)$$

which reduces to the Pennes bioheat equation (1) if $\tau_q = \tau_T = 0$. In absence of phase lag for temperature gradient ($\tau_T = 0$), Eq. (9) is reduced to the hyperbolic conduction model, Eq. (4). Currently, there exists a lot of controversies in the literature about whether or not DPL conduction and, more generally, any non-Fourier conduction is important for biological tissues. Specifically, there is limited experimental evidence for this phenomenon, and some of the non-Fourier evidence has been called into question repeatedly [11,12].

The DPL bioheat equation (9) is obtained by simply modification of the fundamentally questionable Pennes bioheat equation, which is not a convincing approach. Another reason that the DPL bioheat equation is still not widely accepted by the researchers is lacking of appropriate theoretical models on estimation of the two phase lag times. The root of dual-phase lag phenomena in the living biological tissue is nonequilibrium between the blood in artery and the surrounding tissue. In this paper, the DPL bioheat equations will be developed based on the nonequilibrium heat transfer in the living tissue. A significant advantage of the DPL bioheat equation that will be developed in this paper is that the phase lag times can be expressed in terms of the properties of blood and tissue and the interphase convective heat transfer coefficient and blood perfusion rate. The phase lag times for heat flux and temperature gradient are estimated using the available properties from the literature.

2. Nonequilibrium heat transfer in living tissues

Heat transfer in living biological tissues involves heat conduction in tissue, convective heat transfer between blood and vessel, and blood perfusion. The biological tissue can be divided into two regions: vascular region (blood vessel) and extravascular region (tissue). Therefore, the whole anatomical structure can be

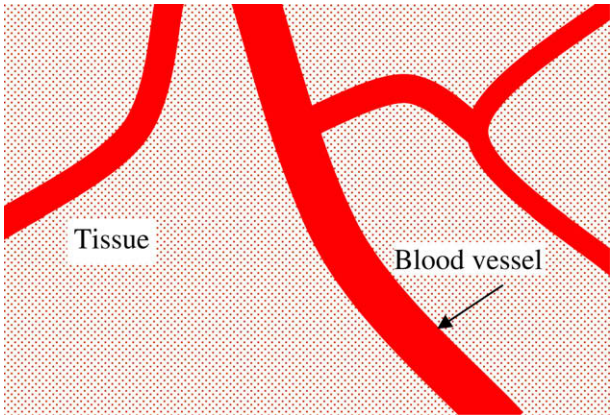

Fig. 1. Schematic of blood vessel and surrounding tissue.

treated as a fluid saturated porous medium (see Fig. 1), i.e., the extravascular region is considered as a solid matrix and blood infiltrates in the pore space of the porous medium [13]. The transport phenomena in the biological tissue can thus be considered as convection in porous media with internal heat generation. Fundamental formulations of the governing equations based on equilibrium between the solid matrix and fluid within a porous media have been presented in the literature [14–17].

For heat transfer in living biological tissues, the temperatures of blood and tissue are different and the equilibrium assumption is invalid. Although the Pennes bioheat equation recognized the different temperatures between the tissues and blood, the blood temperature was assumed to be a constant. In reality, the blood temperature changes as result of convective heat transfer between the blood and tissue and blood perfusion. Xuan and Roetzel [18] performed volume average to the local instantaneous governing equation for blood and tissues obtained a two-temperature model. The following equations are applicable for the case that the internal heat source by hyperthermia therapy is present:

$$\varepsilon\rho_b c_b\left[\frac{\partial T_b}{\partial t}+\mathbf{V}\cdot\nabla T_b\right]=\nabla\cdot(k_{b,eff}\nabla T_s)+a_b h_b(T_s-T_b)+\varepsilon S \tag{10}$$

$$(1-\varepsilon)\rho_s c_s\frac{\partial T_s}{\partial t}=\nabla\cdot(k_{s,eff}\nabla T_s)+a_b h_b(T_b-T_s)+(1-\varepsilon)S_m+(1-\varepsilon)S \tag{11}$$

where the blood and tissue temperatures are volume averaged values, and $k_{b,eff}$ and $k_{s,eff}$ are effective thermal conductivity of blood and the solid matrix (tissue), respectively. Eqs. (10) and (11) include significant effects from the blood flow and direction, thermal diffusion and the local thermal nonequilibrium between the blood and the peripheral tissues. The effects of vascular geometry and size can also be accounted for by the convective heat transfer coefficient h_b and the specific area of the blood vessel in the tissue a_b.

Comparing Eq. (11) with Eq. (1) indicates that the blood perfusion term is replaced by the interfacial convective heat transfer term. The interfacial convective heat transfer and blood perfusion are different processes and should not be confused [13]. The former is caused by temperature difference between the blood and the vessel, regardless if blood perfusion occurs. On the other hand, blood perfusion is the process of nutritive delivery of arterial blood to a capillary bed in the biological tissue, i.e., a mass transfer process. The temperature of the blood is decreased from T_b to T_s in the perfusion process and the energy change in this process can be represented by the blood perfusion term in Eq. (1). Realizing the difference between the convective heat transfer and blood perfusion, Nakayama and Kuwahara [13] presented a rigorous mathematical development based on volume averaging theory and the following set of volume averaged governing equation for bioheat transfer and blood flow are obtained:

$$\varepsilon\rho_b c_b\left[\frac{\partial T_b}{\partial t}+\mathbf{V}\cdot\nabla T_b\right]=\nabla\cdot(k_{b,eff}\nabla T_s)+a_b h_b(T_s-T_b)+w_b c_b(T_s-T_b)+\varepsilon S \tag{12}$$

$$(1-\varepsilon)\rho_s c_s\frac{\partial T_s}{\partial t}=\nabla\cdot(k_{s,eff}\nabla T_s)+a_b h_b(T_b-T_s)+w_b c_b(T_b-T_s)+(1-\varepsilon)S_m+(1-\varepsilon)S \tag{13}$$

where the third terms in the right-hand side of Eqs. (12) and (13) represent the contributions of blood perfusion on the energy balances in blood and tissue. Nakayama and Kuwahara [13] pointed out that the reason that the blood perfusion terms are missing in Eqs. (10) and (11) is that the term describing the transcapillary fluid exchange via arterial-venous anastomoses was not retained in the volume averaging in Ref. [18].

For the purpose of development of DPL bioheat equations, the two-temperature model can be written in the following compact forms:

$$\varepsilon\rho_b c_b\left[\frac{\partial T_b}{\partial t}+\mathbf{V}\cdot\nabla T_b\right]=\varepsilon k_b\nabla^2 T_s+G(T_s-T_b)+\varepsilon S \tag{14}$$

$$(1-\varepsilon)\rho_s c_s\frac{\partial T_s}{\partial t}=(1-\varepsilon)k_s\nabla^2 T_s+G(T_b-T_s)+(1-\varepsilon)S_m+(1-\varepsilon)S \tag{15}$$

where

$$G=a_b h_b+w_b c_b \tag{16}$$

is a lumped convection–perfusion parameter and is referred to as coupling factor between blood and the tissue. In arriving to Eqs. (14) and (15), it is also assumed that the effective thermal conductivities of blood and tissue are constants: $k_{b,eff}=\varepsilon k_b$ and $k_{s,eff}=(1-\varepsilon)k_s$. This treatment is accurate if the thermal conductivities of blood and tissue are close to each other [19].

3. Dual-phase lag model

The dual-phase lag bioheat equation can be obtained by eliminating either tissue or blood temperature from the two-temperature model represented by Eqs. (14) and (15). Adding Eqs. (14) and (15) together yields the following energy equation for the blood saturated tissue:

$$\varepsilon\rho_b c_b\frac{\partial T_b}{\partial t}+(1-\varepsilon)\rho_s c_s\frac{\partial T_s}{\partial t}+\varepsilon\rho_b c_b\mathbf{V}\cdot\nabla T_b=\varepsilon k_b\nabla^2 T_s+(1-\varepsilon)k_s\nabla^2 T_s+(1-\varepsilon)S_m+S \tag{17}$$

Following the assumption by Minkowycz et al. [20], it is hypothesized that before onset of equilibrium, the blood temperature undergoes a transient process defined by:

$$\varepsilon\rho_b c_b\frac{\partial T_b}{\partial t}=G(T_s-T_b) \tag{18}$$

which can be rearranged to obtain

$$T_s=T_b+\frac{\varepsilon\rho_b c_b}{G}\frac{\partial T_b}{\partial t} \tag{19}$$

Substituting Eq. (19) into Eq. (17), the following equation with blood temperature as sole unknown is obtained:

$$\tau_q\frac{\partial^2 T_b}{\partial t^2}+\frac{\partial T_b}{\partial t}+\frac{\varepsilon\rho_b c_b}{(\rho c)_{eff}}\mathbf{V}\cdot\nabla T_b=\alpha_{eff}\left[\nabla^2 T_b+\tau_T\frac{\partial}{\partial t}(\nabla^2 T_b)\right]+\frac{(1-\varepsilon)S_m+S}{(\rho c)_{eff}} \tag{20}$$

where the phase lags for heat flux and temperature gradient are

$$\tau_q = \frac{\varepsilon(1-\varepsilon)\rho_b c_b \rho_s c_s}{G(\rho c)_{eff}} \tag{21}$$

$$\tau_T = \frac{\varepsilon(1-\varepsilon)\rho_b c_b k_s}{G k_{eff}} \tag{22}$$

where

$$(\rho c)_{eff} = \varepsilon\rho_b c_b + (1-\varepsilon)\rho_s c_s \tag{23}$$

$$k_{eff} = \varepsilon k_b + (1-\varepsilon)k_s \tag{24}$$

$$\alpha_{eff} = \frac{k_{eff}}{(\rho c)_{eff}} \tag{25}$$

are effective heat capacity, thermal conductivity, and thermal diffusivity, respectively. Eq. (20) is the DPL bioheat equation with blood temperature as unknown. It can bee seen from Eqs. (21) and (22) that the phase lag times are expressed in terms of the properties of blood and tissue and the coupling factor between the blood and tissue.

To obtain the bioheat equation with tissue temperature as sole unknown, one can substitute Eq. (20) into Eq. (19) to eliminate the blood temperature, i.e.,

$$\tau_q \frac{\partial^2 T_s}{\partial t^2} + \frac{\partial T_s}{\partial t} + \frac{\varepsilon\rho_b c_b}{(\rho c)_{eff}}\mathbf{V}\cdot\nabla T_s = \alpha_{eff}\left[\nabla^2 T_s + \tau_T \frac{\partial}{\partial t}(\nabla^2 T_s)\right] + \frac{(1-\varepsilon)S_m + S}{(\rho c)_{eff}} + \frac{\varepsilon\rho_b c_b}{G(\rho c)_{eff}}\left[(1-\varepsilon)\frac{\partial S_m}{\partial t} + \frac{\partial S}{\partial t}\right] \tag{26}$$

where it is assumed that $\partial\mathbf{V}/\partial t \approx 0$ to simplify the resulting equation. Eq. (26) is the bioheat equation that describes the tissue temperature distribution in the living biological tissue. Although Eq. (26) has the from similar to Eq. (9), the following differences are noted: (a) tissue temperature is the only unknown in the bioheat equation and the blood temperature does not appear, (b) the effect of conduction and convection in the blood was not included in Eq. (9) but these two effects are incorporated in Eq. (26), (c) the effect of blood flow is not necessarily homogeneous because the directional blood flow can be easily accounted for by the convective term on the left-hand side of Eq. (26), and (d) the phase lag times τ_q and τ_T can be obtained from Eqs. (21) and (22).

While Eq. (26) is in the form that can be directly used to obtain the tissue temperature if the blood velocity is known, it will be helpful if it can be casted in the form that is similar to the Pennes bioheat equation. The contribution of blood flow on the temperature distribution is represented by the third term on the left-hand side of Eq. (26) or the second term on the right-hand side of Eq. (15). Since both of these two terms represent the same physical phenomenon, one can expect that [21]

$$-\varepsilon\rho_b c_b \mathbf{V}\cdot\nabla T_s \approx G(T_b - T_s) \tag{27}$$

Substituting Eq. (27) into Eq. (26), the following DPL bioheat equation is obtained

$$\tau_q \frac{\partial^2 T_s}{\partial t^2} + \frac{\partial T_s}{\partial t} = \alpha_{eff}\left[\nabla^2 T_s + \tau_T \frac{\partial}{\partial t}(\nabla^2 T_s)\right] + \frac{G}{(\rho c)_{eff}}(T_b - T_s) + \frac{(1-\varepsilon)S_m + S}{(\rho c)_{eff}} + \frac{\varepsilon\rho_b c_b}{G(\rho c)_{eff}}\left[(1-\varepsilon)\frac{\partial S_m}{\partial t} + \frac{\partial S}{\partial t}\right] \tag{28}$$

which has the similar form to the DPL bioheat equation (9). The differences between the present DPL bioheat equation (28) and the bioheat equation (9) obtained by using Eq. (7) in the classical Pennes bioheat equation include: (a) Eq. (9) considers heat conduction in tissue only but Eq. (26) considered the effect of conduction by both tissue and blood, and (b) the phase lag times τ_q and τ_T can be obtained from Eqs. (21) and (22). Depending on whether the blood velocity is available, either Eq. (26) or (28) are acceptable form of DPL bioheat equations.

4. Phase lag times for living biological tissues

One of the most significant advantages of the DPL bioheat equations (26) or (28) over the DPL bioheat equation (9) is that the phase lag times are expressed in terms of the properties of blood and tissue and the interphase convective heat transfer coefficient and blood perfusion rate. The phase lag times of the living biological tissues will be estimated in this section. Substituting Eqs. (23) and (24) into Eqs. (21) and (22), the phase lag times for heat flux and temperature gradient become

$$\tau_q = \frac{\varepsilon(1-\varepsilon)}{[\varepsilon/C_{sb} + (1-\varepsilon)]}\frac{\rho_b c_b}{G} \tag{29}$$

$$\tau_T = \frac{\varepsilon(1-\varepsilon)}{[\varepsilon/K_{sb} + (1-\varepsilon)]}\frac{\rho_b c_b}{G} \tag{30}$$

where

$$C_{sb} = \rho_s c_s/(\rho_b c_b) \tag{31}$$

$$K_{sb} = k_s/k_b \tag{32}$$

are the ratios of heat capacities and thermal conductivities of tissue and blood. Since the root of dual-phase lag phenomena is the nonequilibrium between blood and tissue, it is evident from Eqs. (29) and (30) that the phase lag times are governed by the porosity, heat capacities of blood and tissues, coupling factor, and the ratio of thermal conductivities of tissue and blood. On the contrary that the thermal conductivity of blood did not appear in the existing DPL models [e.g., Eq. (9)], this paper revealed that the phase lag time for temperature depends on the ratio of thermal conductivities of tissue and blood (but not the individual values of thermal conductivities for tissue and blood).

The coupling factor between blood and tissue, G, appeared in Eqs. (29) and (30) is an very important property of the living tissue because it dictates energy exchange between the blood and surrounding tissues. As indicated by Eq. (16), both convective heat transfer and blood perfusion contribute to the coupling between the blood and tissue. For the bundle of vascular tube with diameter of d_b, the specific area and heat transfer coefficient are $a_b = 4\varepsilon/d_b$ and $h_b = \mathrm{Nu}(k_b/d_b)$, respectively [13]. Therefore, the product of specific area and heat transfer coefficient becomes

$$a_b h_b = \frac{4\varepsilon k_b}{d_b^2}\mathrm{Nu} \tag{33}$$

Substituting Eq. (33) into Eq. (16), the coupling factor becomes

$$G = \frac{4\varepsilon k_b}{d_b^2}\mathrm{Nu} + w_b c_b \tag{34}$$

where the Nusselt number can be approximately set to Nu = 4.93 [22,23]. Therefore, the coupling factor can be obtained if the porosity, diameter of the vascular tube, and blood perfusion rate are available.

Yuan [24] presented a numerical analysis of an equivalent heat transfer coefficient in a porous model for simulating a biological tissue in a hyperthermia therapy based on the nonequilibrium heat transfer model of Ref. [18]. Following the same design parameter as Ref. [25], the blood vessel was assumed to be uniformly distributed in the tissue so that the whole domain was considered to be an assembly of repeated hexagon unit that has an equivalent circle

with a diameter of d_s (see Fig. 2). Under this ideal structure, the distance between two blood vessels is $L = 1.05d_s$, and the porosity of the biological tissue is $\varepsilon = (d_b/d_s)^2$. Different diameters of blood vessels, porosity, and perfusion coefficient are investigated and the parameters are summarized in Table 1.

With the structure and blood perfusion parameters specified in Table 1, the coupling factors and the phase lag times are functions of the thermophysical properties of blood and tissue only. Yuan [24] assumed that the thermophysical properties of blood and tissues are identical: $k_b = k_s = 0.5$W/m K, $\rho_b = \rho_s = 1050$ kg/m^3, and $c_b = c_s = 3770$ J/kg K. The contributions of convective heat transfer and blood perfusion in blood-tissue coupling, coupling factor, and the phase lag times with the above properties are listed in Table 2. It can be seen that while the coupling between the blood and tissue is dominated by contribution of the convective heat transfer, the contribution of blood diffusion on coupling between blood and tissue ranges from 10% to 20%, depending on the blood perfusion rate. This result quantitatively confirms the reasoning by Nakayama and Kuwahara [13] who suggested that the contributions of blood perfusion on the interphase heat transfer is not negligible. The phase lag times for heat flux and temperature gradient are identical ($\tau_q = \tau_T$) because $K_{sb} = C_{sb} = 1$ [see Eq. (29) and (30)]. For the three different blood vessel diameters studied, it can be seen that the phase lag times significantly increase with increasing blood vessel diameter, which means the dual-phase lag phenomenon is more pronounced when large blood vessel is present. For the same blood vessel diameter, the dual-phase lag times slightly decrease as blood perfusion rate increases.

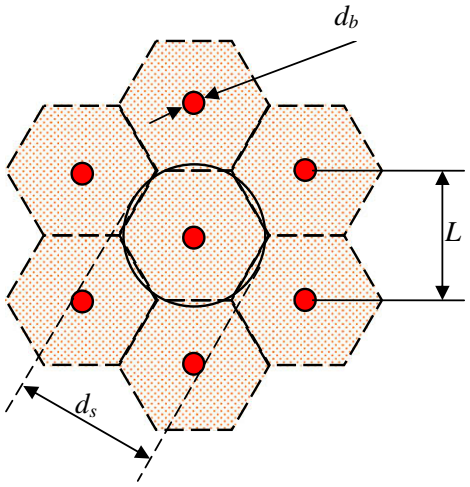

Fig. 2. Hexagonal unit of blood vessel and surrounding tissue.

Table 2
Coupling factor and phase lag times ($k_b = k_s = 0.5$ W/m K, $\rho_b = \rho_s = 1050$ kg/m^3, and $c_b = c_s = 3770$ J/kg K).

Case	$a_b h_b$ (W/m^3 K)	$c_b w_b$ (W/m^3 K)	G (W/m^3 K)	τ_q (s)	τ_T (s)
1	31,094	3770	34,864	0.464	0.464
2	59,895	7540	67,435	0.460	0.460
3	85,540	11,310	96,850	0.457	0.457
4	105,044	15,080	120,124	0.452	0.452
5	132,050	18,850	150,900	0.451	0.451
6	25,158	3770	28,928	1.756	1.756
7	47,538	7540	55,078	1.752	1.752
8	67,960	11,310	79,270	1.733	1.733
9	89,875	15,080	104,955	1.723	1.723
10	100,643	18,850	119,493	1.663	1.663
11	22,472	3770	26,242	6.825	6.825
12	39,948	7540	47,488	6.449	6.449
13	53,592	11,310	64,902	6.127	6.127
14	67,960	15,080	83,040	5.866	5.866
15	77,777	18,850	96,627	5.630	5.630

The thermophysical properties of tissue depend on the type and location of the tissue in the body. Therefore, the assumption of the same thermophysical properties for blood and tissue is not always valid. Evaluation of the dual-phase lag times is then carried out for the biological tissue studied in Ref. [10]: $\rho_s = 1000$ kg/m^3, $k_s = 0.628$ W/m K, $c_s = 4187$ J/kg K, $\rho_b = 1060$ kg/m^3, and $c_b = 3860$ J/kg K. The thermal conductivity for blood was not used in Ref. [10] since the contribution of heat conduction in the blood was considered in Ref. [10]. On the other hand, the thermal conductivity of blood ($k_b = 0.5$W/m K) used by Yuan [24] agreed with other sources [25] and appeared to be reasonable. The diameter of the blood vessel and the porosity of the tissue were not available in Ref. [10] so that values of d_b and ε in Table 1 are reused. The contributions of convective heat transfer and blood perfusion in blood-tissue coupling, coupling factor, and the phase lag times with the properties used in Ref. [10] are listed in Table 3. Similar to Table 2, the coupling between the blood and tissue is dominated by contribution of the convective heat transfer, and the contribution of blood diffusion on coupling between blood and tissue also ranges from 10% to 20%. The ratios of thermal conductivities and heat capacities are $K_{sb} = 1.256$ and $C_{sb} = 1.023$, respectively. For the cases of $d_b = 1.14$ mm, the phase lag times for heat flux and temperature gradient are almost identical and are increased by 3% compared to the cases in Table 2 with the same porosity and perfusion rate. For the cases of $d_b = 2.28$ mm, the phase lag times for temperature gradient are slightly higher than that for the heat flux but the difference is less than 1%. Compared

Table 1
Structures and perfusion coefficient studied in Ref. [23].

Case	d_s (mm)	d_b (mm)	ε	w_b (kg/m^3 s)
1	17.83	1.14	0.0041	1
2	12.85	1.14	0.0079	2
3	10.75	1.14	0.0113	3
4	9.7	1.14	0.0139	4
5	8.65	1.14	0.0175	5
6	19.82	2.28	0.013	1
7	14.42	2.28	0.025	2
8	12.06	2.28	0.036	3
9	10.48	2.28	0.048	4
10	9.92	2.28	0.053	5
11	20.98	4.56	0.0475	1
12	15.73	4.56	0.0845	2
13	13.58	4.56	0.1133	3
14	12.06	4.56	0.1437	4
15	11.27	4.56	0.1645	5

Table 3
Coupling factor and phase lag times ($\rho_s = 1000$ kg/m^3, $k_s = 0.628$ W/m K, $c_s = 4187$ J/kg K, $\rho_b = 1060$ kg/m^3, and $c_b = 3860$ J/kg K).

Case	$a_b h_b$ (W/m^3 K)	$c_b w_b$ (W/m^3 K)	G (W/m^3 K)	τ_q (s)	τ_T (s)
1	31,094	3860	34,954	0.478	0.478
2	59,895	7720	67,615	0.474	0.475
3	85,540	11,580	97,120	0.471	0.472
4	105,044	15,440	120,484	0.466	0.467
5	132,050	19,300	151,350	0.465	0.466
6	25,158	3860	29,018	1.810	1.814
7	47,538	7720	55,258	1.806	1.814
8	67,960	11,580	79,540	1.787	1.798
9	89,875	15,440	105,315	1.777	1.793
10	100,643	19,300	119,943	1.714	1.731
11	22,472	3860	26,332	7.038	7.099
12	39,948	7720	47,668	6.653	6.757
13	53,592	11,580	65,172	6.324	6.456
14	67,960	15,440	83,400	6.057	6.219
15	77,777	19,300	97,077	5.815	5.994

with cases in Table 2 with the same porosity and perfusion rate, the maximum changes of phase lag times for heat flux and temperature are 3% and 4%, respectively. When the diameter of the blood vessel is increased to 4.56 mm, differences between the phase lag times for heat flux and temperature are between 0.9% and 3.1%. Compared with the cases in Table 2 with the same porosity and perfusion rate, the maximum changes of phase lag times for heat flux and temperature are 3% and 6%, respectively.

Although the thermal conductivities and heat capacities of blood and tissue in Table 3 are different ($K_{sb} = 1.256$ and $C_{sb} = 1.023$), the phase lag times for heat flux and temperature gradient are still very close to each other because the maximum difference between τ_q and τ_T, which occurs in Case 15, is only 3.1%. Therefore, one can conclude that the phase lag times of the temperature gradient (τ_T) and the phase lag of the heat flux (τ_q) are almost identical for all parameters studied in this paper. It should be pointed out that this conclusion is valid only under the parameters that used in this paper due to large variation of thermophysical properties and structure of different biological tissues. Zhou et al. [10] have found that the DPL bioheat conduction equation can be reduced to the Fourier heat conduction equation only if both the phase lag times of the temperature gradient (τ_T) and the phase lag of the heat flux (τ_q) are zero. This is different from the DPL model for pure conduction materials, for which it can be reduced to the Fourier's heat conduction model provided that $\tau_q = \tau_T$. Therefore, the dual-phase effect in biological tissue will be important even though $\tau_q \approx \tau_T$.

Three forms of DPL bioheat equations are presented in this paper. While the sole unknown in Eq. (20) is the blood temperature, Eq. (26) is in term of tissue temperature. Applications of these two bioheat equations require the knowledge of the intrinsic averaged velocity vector in the living tissue, which can be obtained by solving continuity and momentum equations in porous media [21]. If the blood temperature distribution is known (e.g., constant), Eq. (28) can be used to determine the tissue temperature.

5. Conclusions

Generalized dual-phase lag bioheat equations are obtained by analyzing nonequilibrium heat transfer in living biological tissue. The dual-phase lag bioheat equations with blood or tissue temperature as sole unknown temperature are obtained by eliminating the tissue or blood temperature from the nonequilibrium model. The phase lag times are then expressed in terms of the properties of blood and tissue and the interphase convective heat transfer coefficient and blood perfusion rate. When the density, specific heat, and thermal conductivities of blood and tissue are the same, the phase lag times for heat flux and temperature gradient are identical ($\tau_q = \tau_T$). The phase lag times significantly increase with increasing blood vessel diameter, which means the dual-phase lag phenomenon is more pronounced when large blood vessel is present. For the same blood vessel diameter, the dual-phase lag times slightly decrease as blood perfusion rate decreases. When the difference between density, specific heat, and thermal conductivities of blood and tissues are considered, the phase lag times for heat flux and temperature gradient are still very close to each other. Therefore, the phase lag times of the temperature gradient (τ_T) and the phase lag of the heat flux (τ_q) are almost identical for all parameters studied in this paper. Since the DPL bioheat conduction equation cannot be reduced to the Fourier heat conduction equation unless both the phase lag times of the temperature gradient (τ_T) and the phase lag of the heat flux (τ_q) are zero, the dual-phase effect in biological tissue will be important even though $\tau_q \approx \tau_T$.

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